5th Canadian Conf. Earthquake Engineering / Ottawa / 1987 / 5ème Conf. Canadienne Gênie Sismique

Seismic analysis of a PWR nuclear containment shell structure

S.C.Das & R.M.Bakeer
Tulane University, New Orleans, La., USA

ABSTRACT: The purpose of this paper is to summarize the analytical method, assumptions, procedures and results of the seismic analysis of a PWR containment structure. The containment is a reinforced concrete structure in the shape of a cylinder with a hemispherical dome and flat foundation slab. Seismic analysis for the containment structure is performed by including rocking and swaying springs in the mathematical model to take into account the effect of soil-structure interaction. Analysis of the model is accomplished by modal superposition method using the response spectra approach. The generated inertia forces in root mean square are used as input loads for the analysis of the containment. The inertia force on each mass is distributed over the surface which the mass represented. The forces are transformed to the Fourier series and are in the form applicable to the Kalnins' shell program to obtain the deformations and stresses.

1 INTRODUCTION

The containment is a reinforced concrete structure in the shape of a cylinder with a hemispherical dome and flat foundation slab. The cylindrical shell is 140 feet inside diameter, 4 feet thick and 156 feet high. The hemispherical dome is 70 feet inside radius and 3 feet thick shell. The foundation slab is 156 feet diameter and 14 feet thick (Fig.1). Seismic analysis for the containment structure was performed by including rocking and swaying springs in the mathematical model to take into account the effect of soil structure interaction. The stored energies in the superstructure and in the swaying and rocking springs are calculated and the weighted damping for first five modes are evaluated. Seismic analysis is then performed by using these weighted damping for the first five modes.

The analysis is based on the response spectra approach in which the earthquake is defined with frequency dependent response spectra curves (Fig.2). For design-basis earthquake (DBE), the ground acceleration of 0.1 g horizontally and 0.067 g vertically are used. For operating basis earthquake (OBE), a factor of 0.5 is used to multiply the response spectra for DBE. Different damping coefficients are used in calculating the weighted damping for these two cases.

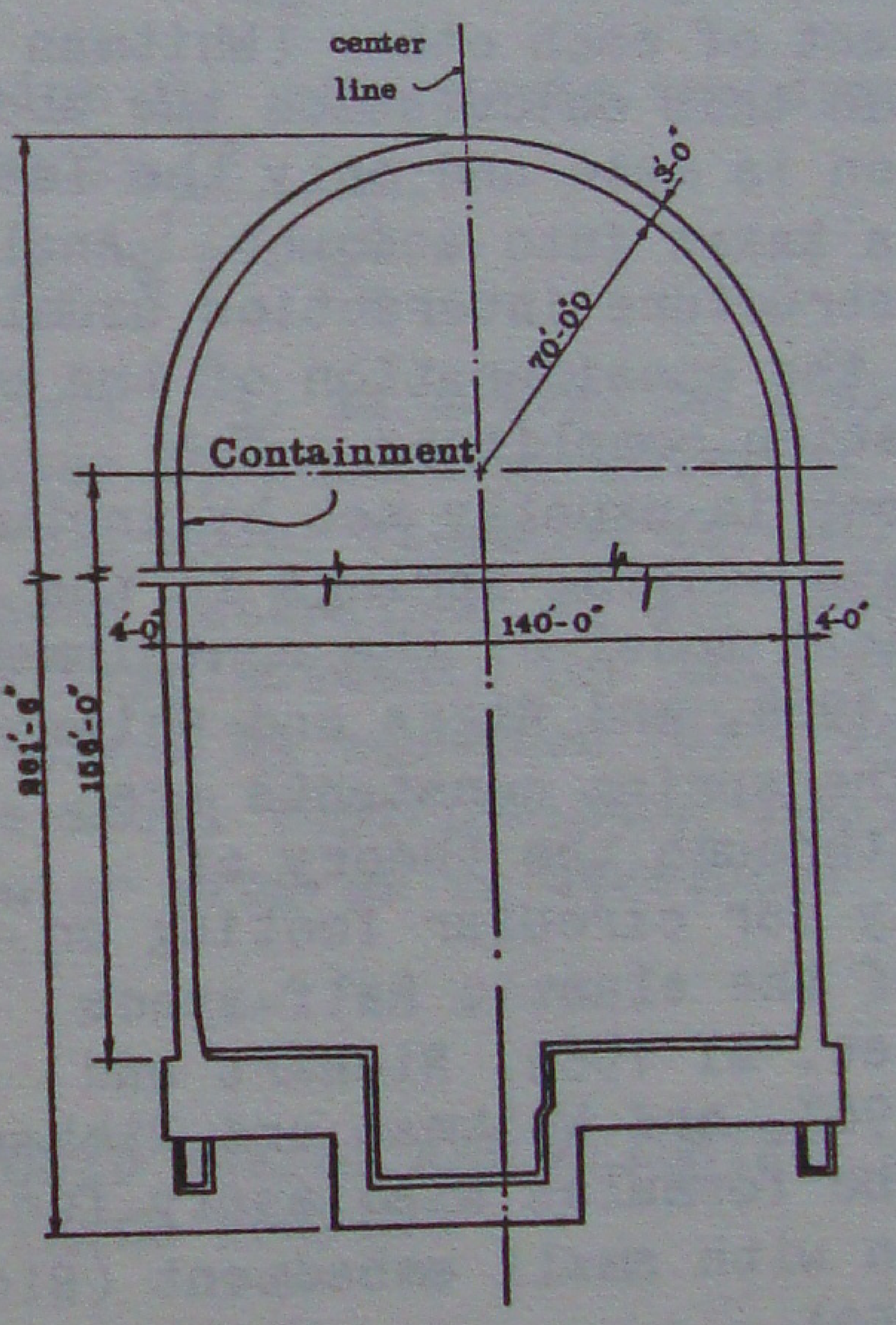


Fig. 1 Prestressed concrete containment.

The generated inertia forces are accepted as input loads for the General Shell Program to obtain the deformation

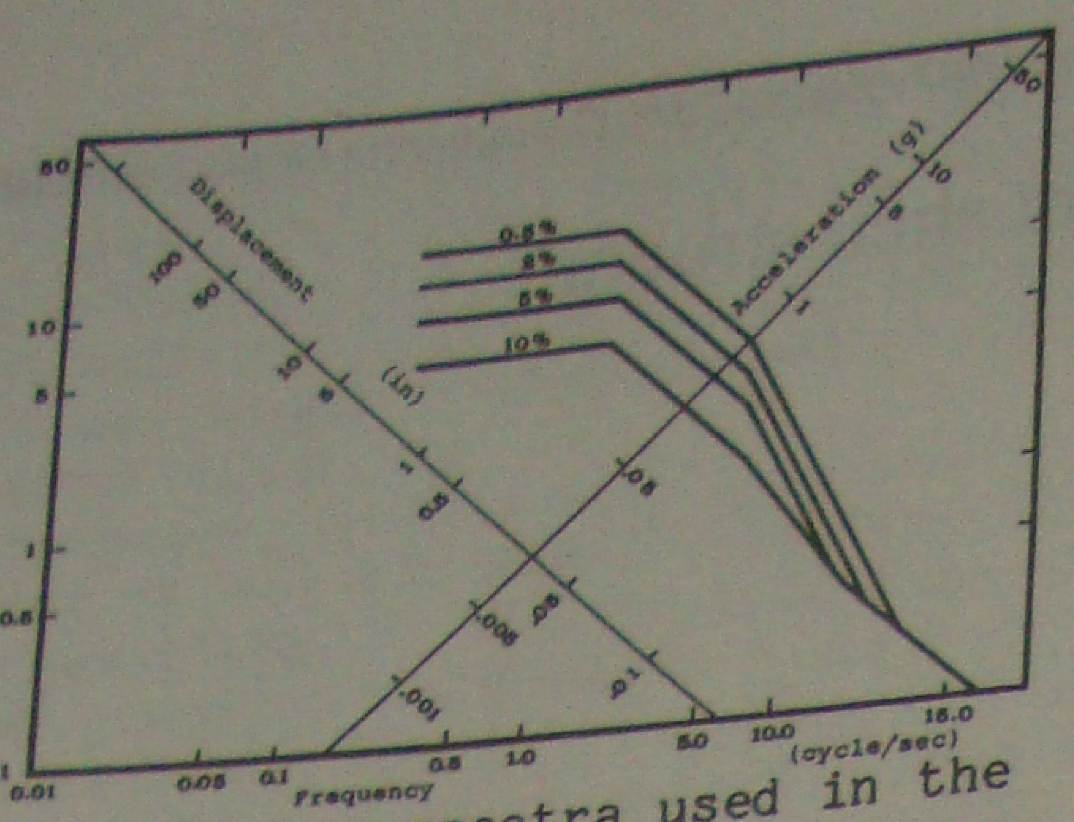


Fig. 2 Response spectra used in the analysis.

and stresses at each parts of the containment.

2 SOIL-STRUCTURE INTERACTION

The soil conditions at the site of a building affect the earthquake response of the building in two ways; soil amplification (Donovan and Matthiesen 1968, and Idriss and Seed 1968) and soil-structure interaction (Whitman 1968). In the case where the depth of the soil is much greater than the width of the building, the effects are independent of each other (Whitman 1970). In this calculation the above assumption is made and only the latter effect is taken into account. Analysis of soil-structure interaction usually requires the consideration of the effect of foundation compliance. The requirement is usually met by including rocking and swaying springs in the mathematical model of the containment (Whitman 1968, and Biggs and Whitman 1970). The spring constants are obtained through the theory of elasticity for circular footing on the surface of the elastic half-space (Richart et. al 1969, Richart and Whitman 1967, and Whitman and Richart 1967). The formulas also apply for mat foundation with small embedment (Richart et. al 1969).

Vertical spring,

$$K_V = \frac{4Gr_o}{1-\nu} \quad (1)$$

(Timoshenko & Goodier 1951)

Horizontal spring,

$$K_H = \frac{32(1-\nu)Gr_o}{7-8\nu} \quad (2)$$

(Byeroft 1956)

Rocking spring,

$$K_R = \frac{8Gr_o^3}{3(1-\nu)} \quad (3)$$

(Borowicka 1943)

3 MATHEMATICAL CODE

The model adopted for the containment dynamic analysis consists of two individual cantilevers representing the containment and internal structure respectively. The two cantilever are founded on the same base which, in turn, is supported by a vertical spring, swaying spring and rocking spring as calculated from the equations (1), (2), and (3) respectively due to the soil-structure interactions.

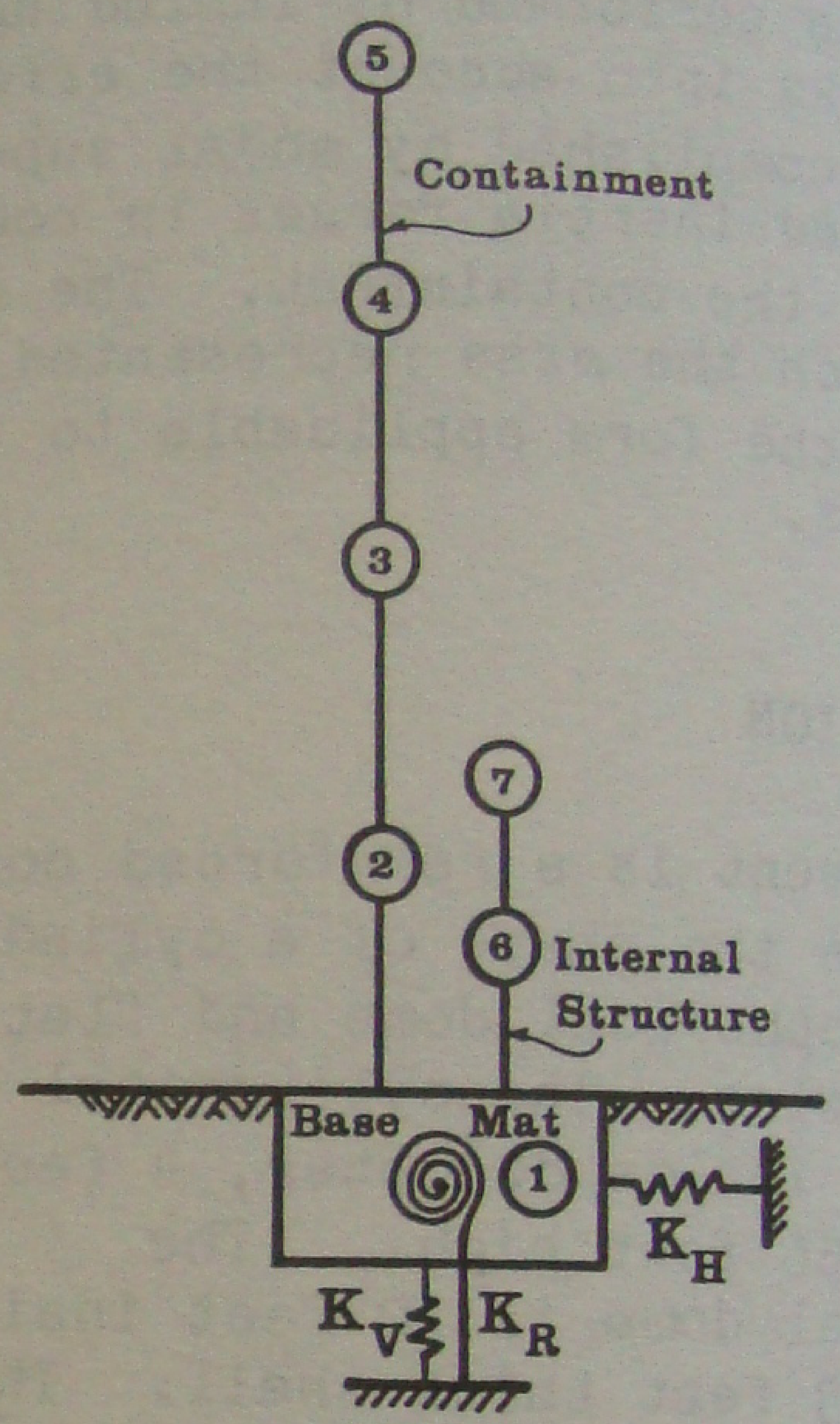


Fig. 3 Mathematical model.

In this idealization, the mass properties of the system are separated from the elastic properties and equivalent concentrated masses are placed at the nodal points to represent the inertia forces in the direction of the assumed element degrees of freedom. These masses refer to both translational and rotational inertia of the element. Only nominal number of masses are used to simulate the structure. It is essential that the mathematical model must faithfully represent all essential characteristics of the distribution of mass and of stiffness in the structure. For the PWR containment under study, the ratio of the height to the width is only about 1.5. When this structure is represented by a one dimensional model, it makes no sense to use numerous masses representation especially for the preliminary study while the geological formation of the site and the precise nature of future ground motion are still unknown. The dynamic response analysis of a system is performed by solving the equations of motion;

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P(t)\} \quad (4)$$

where,

$[M]$ = mass matrix
$[C]$ = damping matrix
$[K]$ = stiffness matrix
$\{P(t)\}$ = forcing function
$\{u\}$, $\{\dot{u}\}$, and $\{\ddot{u}\}$ = time dependent displacement, velocity and acceleration vectors, respectively.

4 METHOD OF SOLUTION

For a multi-degree of freedom linear elastic structure subjected to base excitation, the equation of motion for the system is expressed as:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}= -[M]\{P\}\ddot{Y}_s(t) \quad (5)$$

where $\ddot{Y}_s(t)$ = the time dependent prescribed reference support acceleration; $\{P\}$ = the column matrix with components of ratio of support acceleration along the direction of displacement u_i to the reference acceleration $\ddot{Y}_s$. The linear transformation is given by,

$$\{u\} = [q]\ \{x\} \quad (6)$$

where $[q]$ is a square matrix formed as an array of the successive mode shape vectors and $\{x\}$ represents the vector of modal amplitudes. Introducing (6) into the equation of motion, (Equation (5)) leads to,

$$[M][q]\{\ddot{x}\}+[C][q]\{\dot{x}\}+[K][q]\{x\}= -[M]\{p\}\ddot{Y}_s(t) \quad (7)$$

Premultiply (7) by the transpose of an arbitrary modal vector $[q]^T$ and taking advantage of the orthogonality properties:

$$[q]^T[M][q] = [M^*]$$
$$[q]^T[K][q] = [K^*] \quad (8)$$
$$[q]^T[C][q] = [C^*]$$

results in uncoupled equation of motion,

$$[M^*]\{\ddot{x}\}+[C^*]\{\dot{x}\}+[K^*]\{x\}= -[q]^T[M]\{p\}\ddot{Y}_s(t) \quad (9)$$

Dividing (9) by generalized mass $[M^*]$ and introducing the alternate definition of $[C^*]$ and $[K^*]$, we obtain the modal equation:

$$\{\ddot{x}_r\}+[2d_r\omega]\{\dot{x}_r\} + [\omega_r^2]\{x_r\} = -[q]^T[M]\{p\}\ddot{Y}_s(t)/[M^*] \quad (10)$$

and the relations:

$$[K^*] = [\omega_r^2]\ [M^*]$$
$$[C^*] = [2d_r\omega_r]\ [M^*]$$

has been used where $d_r (r=1,2,\ldots,n)$ are modal damping factors. The participation factors for the modes are given by:

$$\{\Gamma\} = [q]^T[M]\{P\}/[M^*] \quad (11)$$

Thus, the equation of motion for r^{th} mode is

$$\ddot{x}_r + 2d_r\omega_r\dot{x}_r + \omega_r^2 x_r = -\Gamma_r\ddot{Y}_s \quad (12)$$

The solution of each modal response equation (12) may be performed by the Duhamel integral:

$$x_r(t)= -\frac{1}{f_{D_r}} \int_0^t (\Gamma_r\ddot{Y}_s(\tau)e^{-d_r\omega_r(t-r)} \sin f_{D_r}(t-\tau)d\tau) \quad (13)$$

in which f_{D_r} represents the damped frequency

$$f_{D_r} = \omega_r\sqrt{1-d_r^2} \quad (14)$$

Equation (13) is solved on a digital computer by numerical integral schemes. When the modal response for all modes has been determined at any time "t", the nodal displacements for this time are then given by equation (6). For the above modal superposition analysis a computer program is developed (Kalnins 1968).

5 STORED ENERGY AND WEIGHTED AVERAGE DAMPING

In many practical problems, sufficient accuracy may be obtained using modal superposition together with weighted modal damping (Biggs and Whitman 1970, and Roesset and Whitman 1972). The weighted damping is calculated for each mode according to the stored energy in each spring. The formula reads as follows:

$$D_n = \frac{D_s E_{sn} + D_H E_{Hn} + D_R E_{Rn}}{E_{sn} + E_{Hn} + E_{Rn}} \quad (15)$$

where

D_n = the weighted average damping for the n^{th} mode.

D_s, D_H and D_R = the damping ratios for superstructure, for swaying and for rocking respectively.

and, E_{sn}, E_{Hn} and E_{Rn} = the energies stored in the superstructure, in the swaying and in the rocking springs, respectively in the n^{th} mode.

Table 1 shows the stored energies in the soil-structure and weighted damping for first five modes. The first mode involves primarily rocking and little energy is stored in the swaying spring in this mode. Second and third modes involve a combination of swaying and structural deformation with relatively little rocking. Higher mode consist mostly of structural deformation. This situation is typical for stiff containment buildings founded upon soil.

Table 1. Stored Energy and Weighted Damping.

Mode	Frequency (CPS)	Percentage Energy Super Structure	Swaying Spring	Rocking Spring	Weighted Damping (%)
1	2.377	26.3	18.5	56.2	6.582
2	4.496	67.6	27.9	4.50	7.792
3	6.704	43.8	48.2	8.00	9.825
4	12.939	94.9	1.80	3.30	5.180
5	14.973	96.2	0.0	3.80	5.007

Weighted Damping = $\frac{D_s E_{sn} + D_H E_{Hn} + D_R E_{Rn}}{E_{sn} + E_{Hn} + E_{Rn}}$

Values Used: D_s = 5%, D_H = 15%, D_R = 5%

6 EARTHQUAKE SHELL ANALYSIS

The output from the computer program (Brown & Root 1973) for the mathematical model of Fig. 3 gives acceleration, inertia force, shear force and bending moment at each mass level in root mean square. The inertia force on each mass is distributed over the surface which the mass represents as shown on Fig. 4. Thus, the surface forces as shown on Fig. 5 through Fig. 7 are obtained. The surface forces are transformed to the Fourier series expression and are in the form applicable to the Kalnins' shell program (Kalnins 1968). The

* Inertia force on each mass is distributed over the surface which the mass represents.

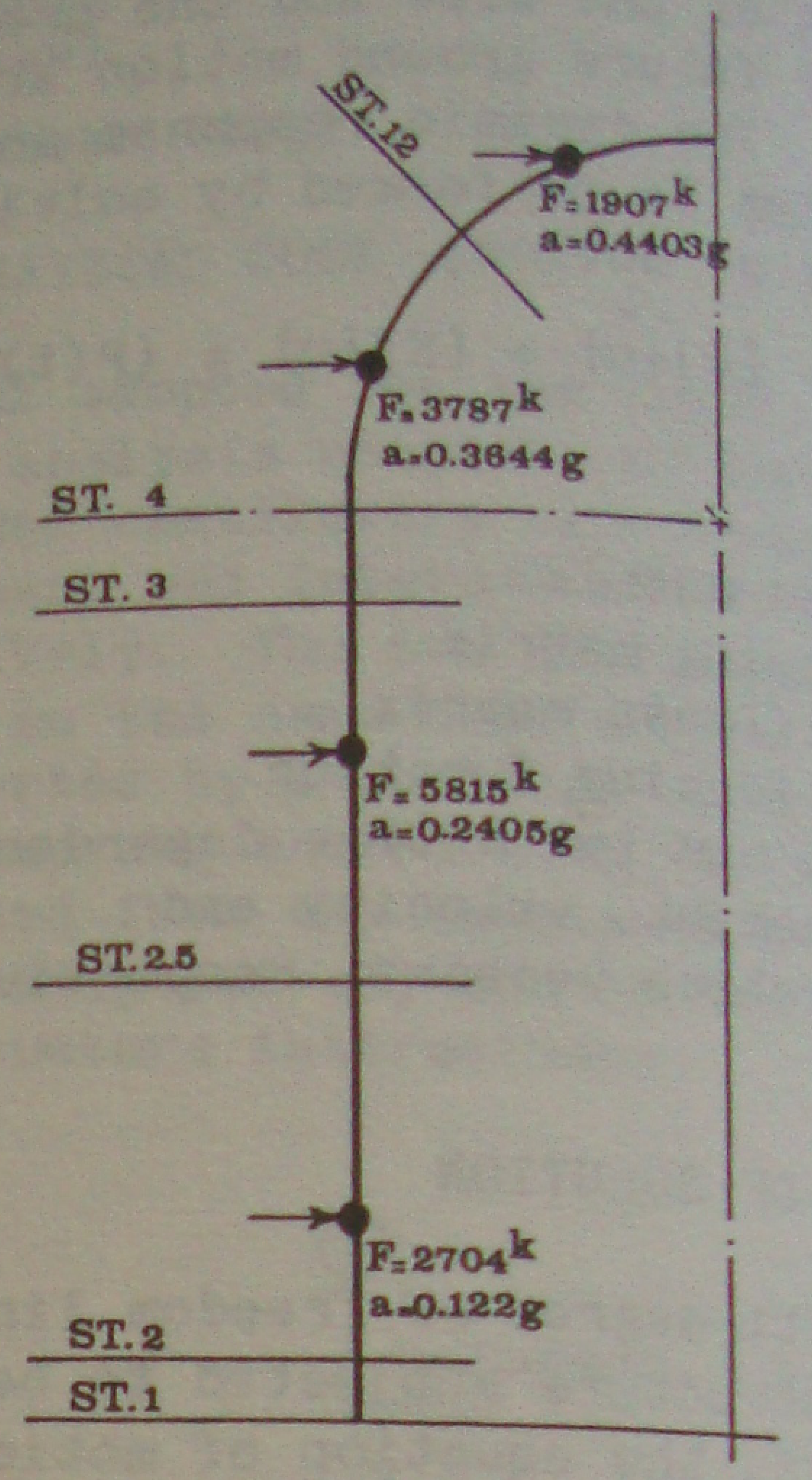


Fig. 4 Inertia forces for DBE.

Cylindrical Parts:

Station 1 thru Station 2.5
Pressure, $f_1 = 2704/(2 \times 72 \times \pi \times 75) = 0.0707$ ksf

Station 2.5 thru 4
Pressure, $f_2 = 5815/(2 \times 72 \times \pi \times 81) = 0.1587$ ksf

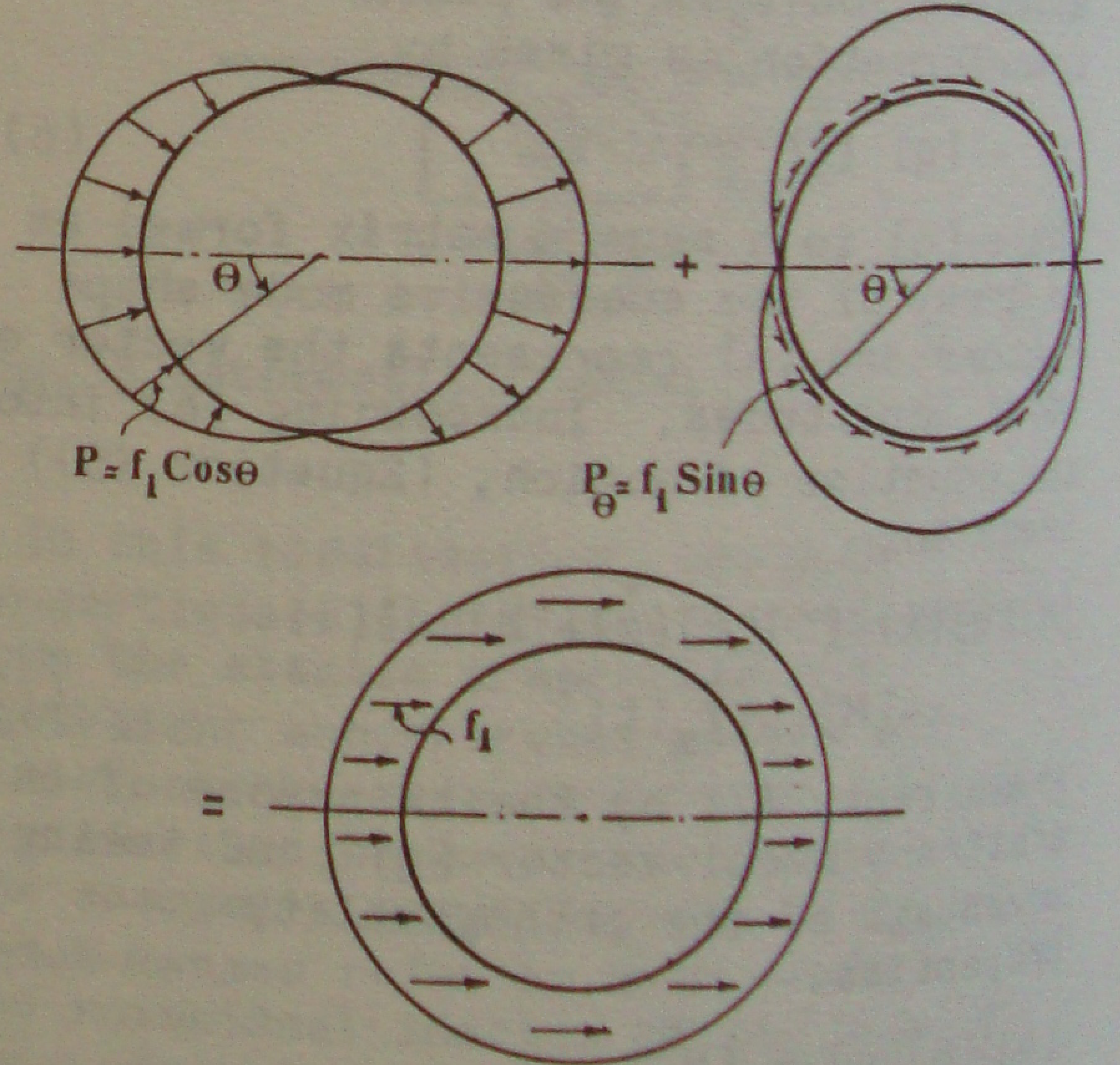
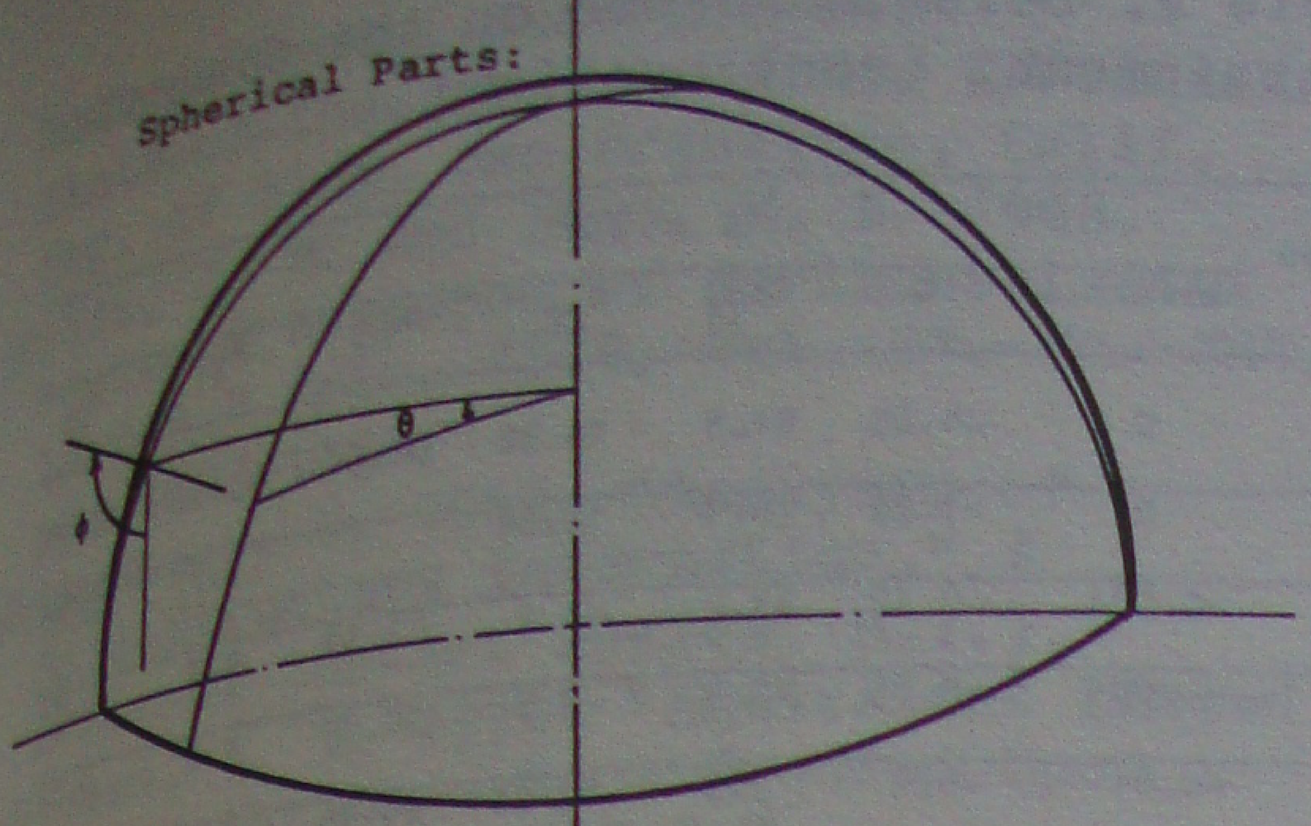


Fig. 5 Fourier coefficients for DBE.

containment structure analytical model used in the program is shown on Fig. 8. The shell material has been assumed to be isotropic and perfectly elastic. Concrete properties used: f'_c = 5000 psi; $E_c = 4.0 \times 10^6$ psi; Poisson's ratio



Station 4 thru Station 12

$$f_3 = 3787/\int_{\pi/4}^{\pi/2}\int_{0}^{2\pi} R^2 \sin\phi\, d\phi\, d\theta = 0.1668 \text{ ksf}$$

Station 12 thru Station 18

$$f_4 = 1907/\int_{0}^{\pi/4}\int_{0}^{2\pi} R^2 \sin\phi\, d\phi\, d\theta = 0.2026 \text{ ksf}$$

Fig. 6 Fourier coefficients for DBE.

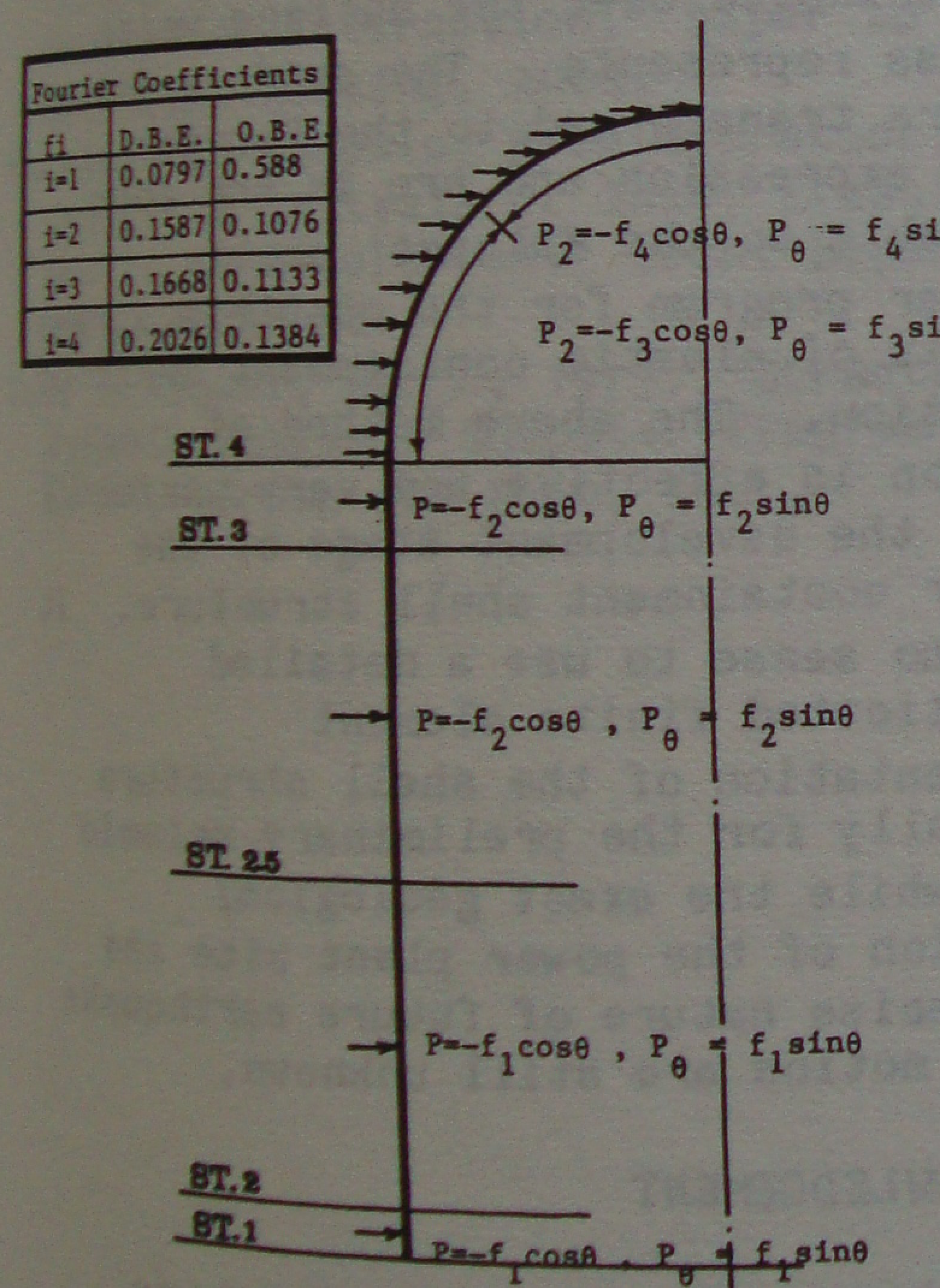


Fig. 7 Seismic loads.

= 0.2 and weight of reinforced concrete = 0.15 K/ft.^{3}. Properties of reinforcing steel used: f_y = 60,000 psi, $E_s = 29 \times 10^6$ psi and weight of steel = 0.489 K/ft^3. Internal forces on a shell element is shown in Fig. 9 and the results of the shell analysis are shown in Table 2 and Fig. 10.

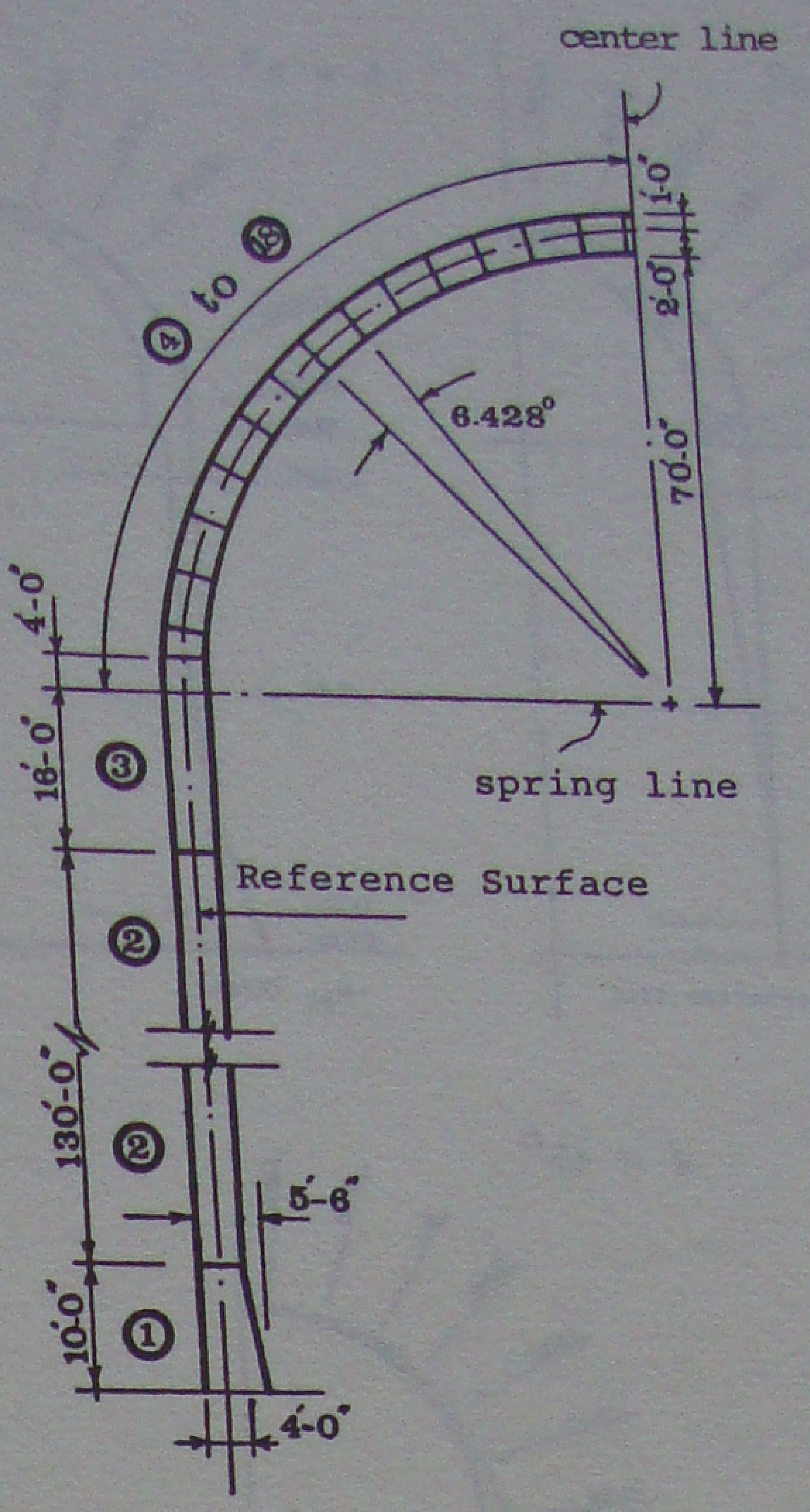


Fig. 8 Analytical model.

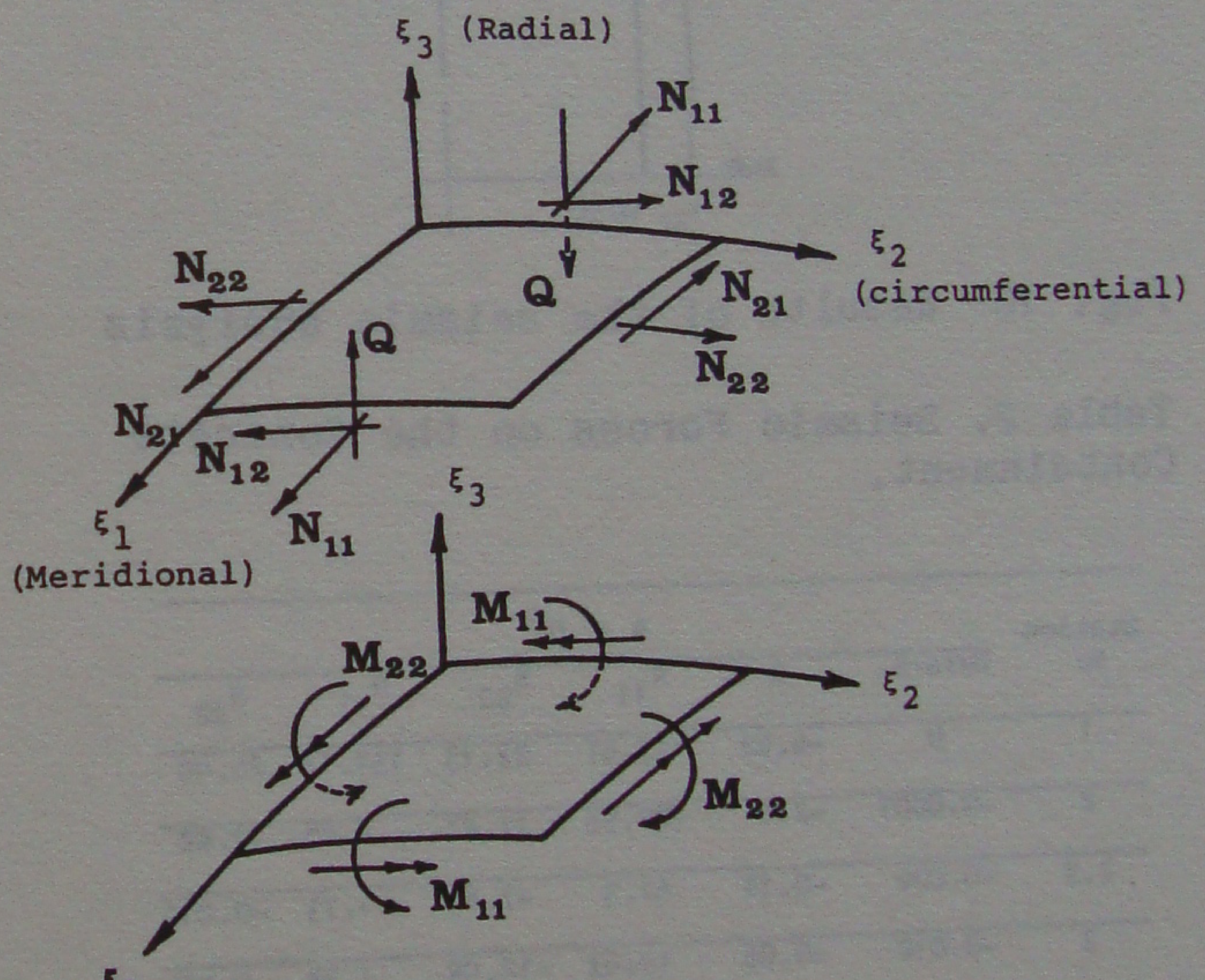


Fig. 9 Internal forces on a shell element.

7 CONCLUSIONS

The computer solution of the simplified lumped mass model of the PWR containment shell structure gives acceleration inertia force, shear force and bending moment at each mass level in root mean square. The inertia force on each mass

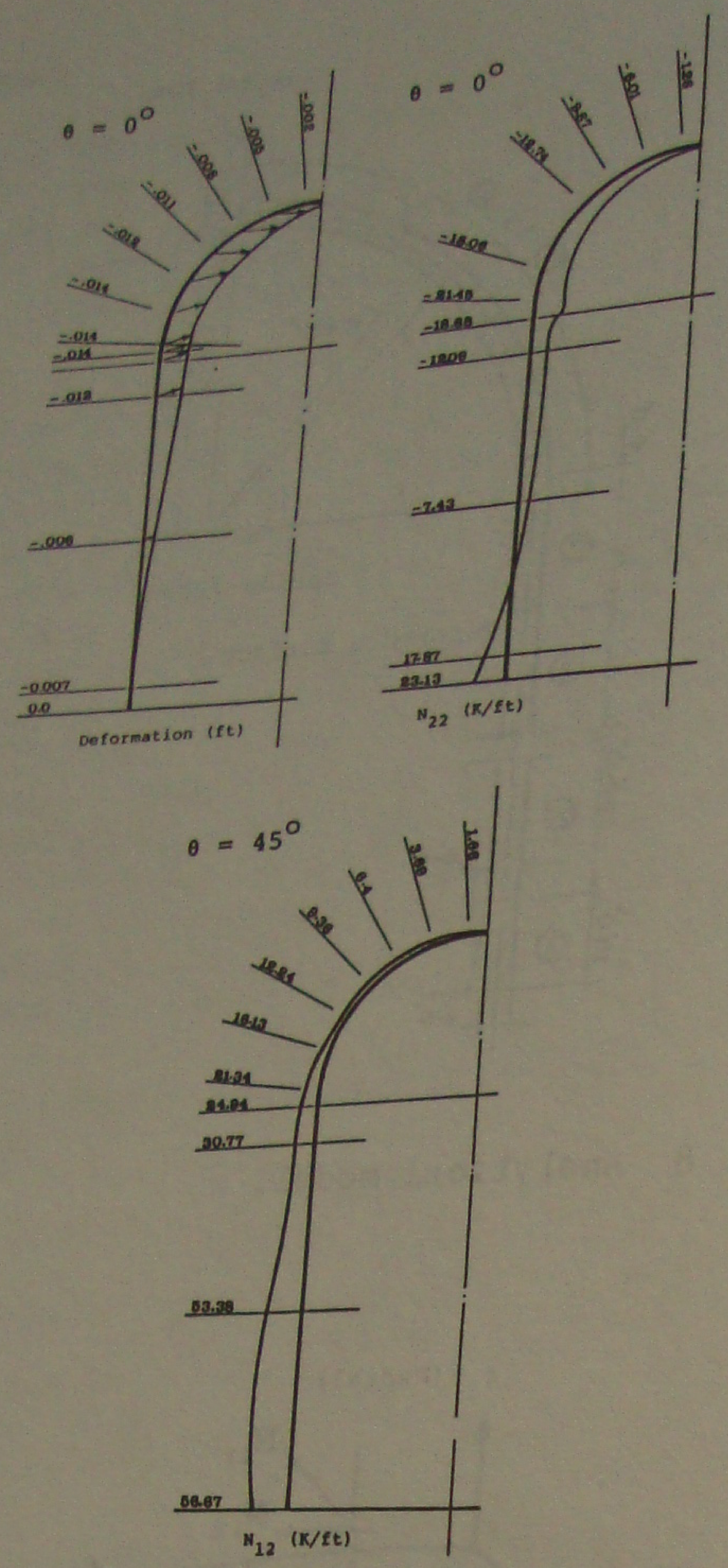


Fig. 10 Results of the seismic analysis

Table 2. Seismic Forces on the Concrete Containment.

Station No	θ = 0°					
	Deform.	Q	N_{11}	N_{22}	M_{11}	M_{22}
1	0	-6.62	115.67	23.13	132.13	26.48
2	-0.0007	-2.6	107.58	17.87	6.65	12.62
2.5	-0.006	-0.14	61.3	-7.43	1.73	-0.019
3	-0.012	-0.06	19.63	-12.09	1.74	0.05
4	-0.014	-0.69	13.41	-18.68	0.48	-0.48
5	-0.014	-0.5	12.04	-21.51	-3.33	-1.21
6	-0.014	-0.39	10.86	-21.45	-3.0	-1.44
8	-0.014		7.41	-18.06	-2.39	
10	-0.012	-0.01	5.16			-0.4
12	-0.011		3.36	-12.74	-0.29	
14	-0.008	0.02	2.27	-9.87		-0.3
16	-0.005		1.21	-6.01	-0.18	-0.12
18	-0.002	0.05	-0.4	-1.26		

Deformation ft
Forces in K/ft
Moments in ft-K/ft

Table 2. Seismic Forces on the Concrete Containment. (continuation)

Station No	θ = 45°						
	Deform.	Q	N_{11}	N_{22}	N_{12}	M_{11}	M_{22}
1	0	-4.68	81.8	16.36	56.67	93.39	18.71
2	0	-1.83	76.1	12.64		4.7	0.89
2.5	-0.004	-0.12	43.0	-5.2	53.35	1.2	-0.01
3	-0.009	-0.05	13.9	-8.5	30.77	1.2	0.04
4	-0.01	-0.4	9.5	-13.21	24.94	-0.34	-0.34
5	-0.01	-0.35	8.5	-15.21	23.03	-2.34	-0.81
6	-0.01	-0.27	7.7	-15.17	21.34	-2.12	-1.01
8	-0.01	-0.02	5.2	-12.77	16.13		
10	-0.009				12.24	-0.02	-0.19
12	-0.007	-0.07	2.6	-6.98	9.36		
14	-0.006				6.4	-0.03	-0.21
16	-0.005	-0.03	1.2	-4.25	3.69	-0.04	
18	-0.001				1.66		0

Deformation ft
Forces in K/ft
Moments in ft-K/ft

is distributed over the surface which the mass represents. The surface forces then are transformed to the Fourier series expression and are in the form applicable to the Kalnins' shell computer program for the complete stress analysis of elastic containment shell of revolution. The above method of solution is effective and very useful during the development stage of the nuclear containment shell structure. It makes no sense to use a detailed sophisticated finite element representation of the shell structure especially for the preliminary seismic study while the exact geological formation of the power plant site and the precise nature of future earthquake ground motion are still unknown.

8 ACKNOWLEDGEMENT

The work described in this paper was mostly performed along with other engineers while the senior author was working with the Brown & Root, Inc., Houston, Texas, during the years 1972-73 for the development stage of south Texas project.

9 REFERENCES

Donovan, N. C. and Matthiesen. 1968. "Effects of Site Conditions on Ground

Motions During Earthquakes," State-of-the-Art Symposium, Earthquake Eng. of Bldg. San Francisco, Calif.

Idriss, I. M. and Seed, H. B. 1968. "Seismic Response of Horizontal Soil Layers." Proc. ASCE, Vol. 94, No. SM4, pp. 1003-1031.

Whitman, R. V. 1968. "Analysis of Soil-Structure Interaction," A State-of-the-Art Review. M.I.T.

Whitman, R. V. 1970. "Equivalent Lamped System for Structure Founded Upon Stratum of Soil." M.I.T.

Biggs, J. M. and Whitman, R. V. 1970. "Soil-Structure Interaction in Nuclear Power Plants." M.I.T.

Richart, F. E. Jr., Hall, J. R. Jr. and Woods, R. D. 1969. "Vibration of Soils and Foundations." Prentice-Hall.

Richart, F. E. Jr. and Whitman, R. V. 1967. "Comparison of Footings Vibration Tests with Theory." Proc. ASCE Vol. 93, No. SM6, pp. 143-168.

Whitman R. V. and Richart, F. E. Jr. 1967. "Design Procedures for Dynamically Loaded Foundations," Proc. ASCE, Vol. 93, No. SM6, pp. 169-193.

Roesset, J. M. and Whitman, R. V. 1972. "Modal Analysis for Structures with Foundation Interaction," paper to Cleveland Meeting, ASCE.

Kalnins, A. "Static, Free Vibration, and Stability Analysis of Thin, Elastic Shells of Revolation." Tech. Report AFFDL-TR-68-144.

User's Manual, March, 1973. "Structural Dynamic Analysis Computer Program - ES414," Brown and Root, Inc.,